



Workshop Aims

Introduction

Key Concepts

Components of a  
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Additive vs  
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Stationarity, ACF  
and PACF

ARIMA Models

Differencing

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Recap

# Quantitative Social Research II

## Workshop 9: Time-Series

Jose Pina-Sánchez

# Workshop Aims

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- Introduce key concepts used to describe time-series
  - Trend, seasonality, irregular component
  - Stationarity
  - Auto-correlation and partial auto-correlation
- Introduce ARIMA models
  - Autoregressive (AR)
  - Moving average (MA)
  - Autoregressive, integration, moving average (ARIMA)
- Learn to forecast and estimate the causal impact of discrete interventions



## What Are Time-Series?

- Any metric that is measured over regular time intervals makes a Time Series
- Share the time dimension that we have seen in longitudinal data
  - But the focus is on one (or a few) subjects/entities
  - Across a much larger timespan
  - The focus tends to be on forecasting based on observed past patterns
  - The modelling strategies are very different

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## What Are Time-Series?

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  - The focus tends to be on forecasting based on observed past patterns
  - The modelling strategies are very different
- Predominantly employed in Economics and by business analysts
  - To forecasts stock prices, changes in GDP
  - To understand economic cycles, or seasonal sales of certain products
- Although less frequent, also used in Sociology and Criminology
  - Estimate the deterrent effect of new (more punitive) laws
  - Estimate the extent to which public demonstrations are seasonally determined

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# Components of a Time-Series

- A time series can be broken down into its main components
  - This helps understand, analyse, and model it
  - And carry out forecasts

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# Components of a Time-Series

- A time series can be broken down into its main components
  - This helps understand, analyse, and model it
  - And carry out forecasts
- Each data point ( $Y_t$ ) at time  $t$  in a time series can be expressed as the combination of
  - Trend ( $T_t$ ): the general tendency of a time series to increase, decrease or stagnate over a long period of time
  - Seasonality ( $S_t$ ): fluctuations within a regular period of time; Ciclicity ( $C_t$ ) can also be included to capture wider fluctuations
  - Irregular component ( $I_t$ ): random variations, do not show a particular pattern, unpredictable

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  - Irregular component ( $I_t$ ): random variations, do not show a particular pattern, unpredictable
- Combined as either a sum or a product
  - Additive model:  $Y_t = T_t + S_t + I_t$   
assumes components are independent of each other
  - Multiplicative model:  $Y_t = T_t \times S_t \times I_t$   
assumes components can affect one another

# Decomposition of Time-Series

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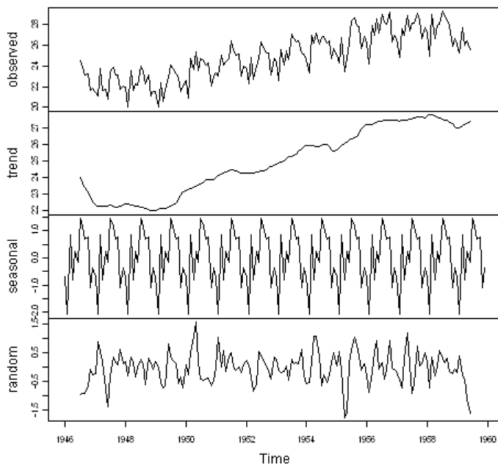
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New York Birth Time-Series (decomposed)

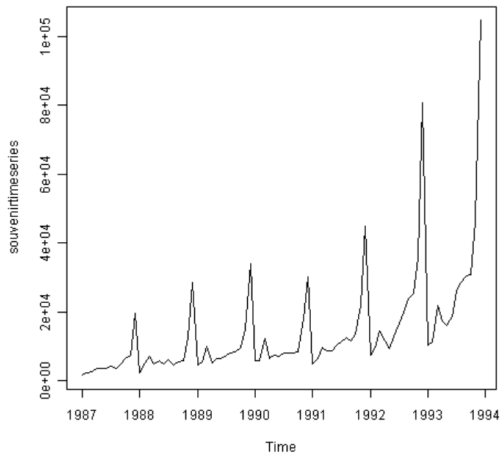


Source: Avril Coghlan



# Multiplicative Model

Monthly sales for a souvenir shop at a beach resort town in Queensland



Source: Avril Coghlan

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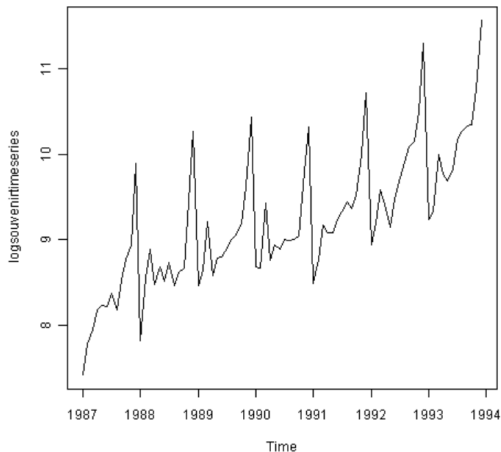
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# Additive Model

Log of monthly sales for a souvenir shop at a beach resort town in Queensland



Source: Avril Coghlan

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- A time-series is said to be stationary if it holds the following conditions true:
  - The mean value of time-series is constant over time, i.e. the trend component is nullified
  - The variance does not increase over time
  - Seasonality effect is minimal
- It looks like random white noise irrespective of the observed time interval



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  - The mean value of time-series is constant over time, i.e. the trend component is nullified
  - The variance does not increase over time
  - Seasonality effect is minimal
- It looks like random white noise irrespective of the observed time interval
- In essence, stationarity means that the statistical properties of a process generating a time series do not change over time
  - To be able to model time-series we are going to seek to turn them stationary



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## ACF and PACF

- To help us decide the most appropriate model we are going to rely on two useful diagnostic functions
- Auto-correlation function (ACF)
  - Gives us values of auto-correlation of any series with its lagged values
  - It describes how well the present value of the series is related to its past values
  - It can be used to inform the number of lags to be included in the model
  - ACF considers all the components in a time-series ( $T$ ,  $S$ ,  $C$  and  $I$ ) in the calculation of its correlations
  - That is why it is known as a ‘complete auto-correlation plot’

## ACF and PACF

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  - ACF considers all the components in a time-series ( $T$ ,  $S$ ,  $C$  and  $I$ ) in the calculation of its correlations
  - That is why it is known as a ‘complete auto-correlation plot’
- Partial auto-correlation function (PACF)
  - Instead of finding correlations of present values with lags, PACF finds correlation of the residuals (remaining after removing the effects explained by the earlier lags) with the next lag value
  - ‘Partial’ because we remove already found variations before calculating the correlation
  - Can help us inform the model if there is any ‘hidden information’ in the residual which can be modeled by the next lag

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- The most common approach to model time-series
- Composed of an autoregressive (AR) part
  - Associated with the ACF
- And a moving average (MA) part
  - Associated with the PACF



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- The most common approach to model time-series
- Composed of an autoregressive (AR) part
  - Associated with the ACF
- And a moving average (MA) part
  - Associated with the PACF
- Requires the time-series to be stationary
  - To do so we can use differencing, aka integration (I)
  - Which stands for the I in ARIMA





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- By subtracting each data point in the series from its successor we can often turn a non-stationary time series stationary

- $Y_t^d = Y_t - Y_{t-1}$

- Useful to remove trends and cycles



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- By subtracting each data point in the series from its successor we can often turn a non-stationary time series stationary

- $Y_t^d = Y_t - Y_{t-1}$

- Useful to remove trends and cycles

- Sometimes higher order differences are necessary to achieve stationarity

- Often a second order difference is enough

- $Y_t^{d2} = Y_t^d - Y_{t-1}^d = (Y_t - Y_{t-1}) - (Y_{t-1} - Y_{t-2})$



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- By subtracting each data point in the series from its successor we can often turn a non-stationary time series stationary
  - $Y_t^d = Y_t - Y_{t-1}$
  - Useful to remove trends and cycles
- Sometimes higher order differences are necessary to achieve stationarity
  - Often a second order difference is enough
  - $Y_t^{d2} = Y_t^d - Y_{t-1}^d = (Y_t - Y_{t-1}) - (Y_{t-1} - Y_{t-2})$
- If the time series appears to be seasonal, a better approach is to difference with respective season's data points
  - $Y_t^d = (Y_t - Y_{t-s}) - (Y_{t-1} - Y_{t-s-1})$
  - This can help to remove the seasonal effect

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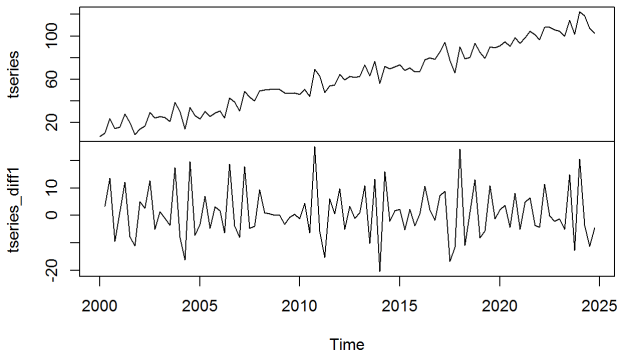
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## Stationary vs Non-stationary

Source: Troy Walters

# Auto-Regressive Models

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- AR are linear models where the outcome variable is regressed on its own lagged values
  - Last week we saw an example of a one order AR, where we also included other explanatory variables
  - Here we will only include lagged values of the outcome as explanatory variables and will often include more than one lag
  - $Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + e_t$
  - If lag up to  $p$  is included in the model, the AR process is said to be of order  $p$



# Auto-Regressive Models

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  - $Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + e_t$
  - If lag up to  $p$  is included in the model, the AR process is said to be of order  $p$
- To decide the order of the AR the model we can use the ACF
  - Which plots the level of auto-correlation at each lag
  - And the 95% confidence interval to determine their statistical significance



# Auto-Correlation Function

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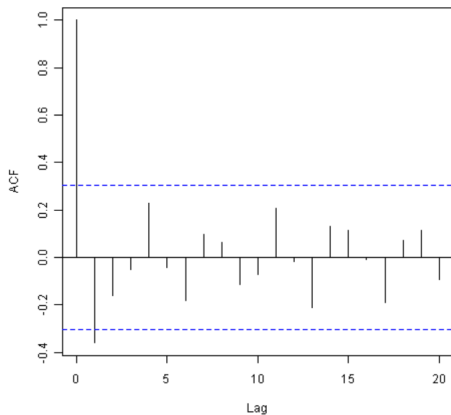
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Source: Avril Coghlan

## Moving Average Models

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- MA are also linear models where the outcome variables is regressed on its own imperfectly predicted lagged values
  - $Y_t = \theta_0 + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \dots + \theta_q e_{t-q} + e_t$
  - If lag up to  $q$  is included in the model, the MA process is said to be of order  $q$
- To decide the order of the MA the model we can use the PACF
  - Which plots the level of auto-correlation at each lag
  - And the 95% confidence interval to determine their statistical significance



# Partial Auto-Correlation Function

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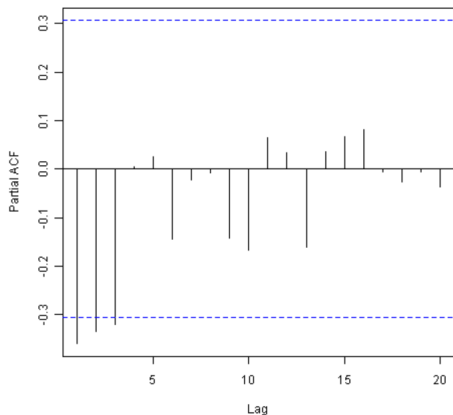
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Source: Avril Coghlan

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- An ARIMA( $p, d, q$ ) model is then defined as the combination of
  - $d$  integrations (differentiations)
  - an AR model of order  $p$
  - and a MA model of order  $q$
- Once the series is turn stationary following the integration process, we have an ARMA( $p, q$ ) model
  - $Y_t = \delta + \{\phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p}\} + \{\theta_1 e_{t-1} + \theta_2 e_{t-2} + \dots + \theta_q e_{t-q}\} + e_t$
  - This is the model to be estimated
  - Which can be used use to make predictions

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- Once we have estimated our ARIMA model we can use it to make predictions
  - Let's assume a simple ARMA(1,1) model
  - $Y_t = \delta + \phi_1 Y_{t-1} + \theta_1 e_{t-1} + e_t$

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- Once we have estimated our ARIMA model we can use it to make predictions
  - Let's assume a simple ARMA(1,1) model
  - $Y_t = \delta + \phi_1 Y_{t-1} + \theta_1 e_{t-1} + e_t$
- We can estimate future values of  $Y$ 
  - This is done sequentially

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- Let's assume a simple ARMA(1,1) model

- $Y_t = \delta + \phi_1 Y_{t-1} + \theta_1 e_{t-1} + e_t$

- We can estimate future values of  $Y$

- This is done sequentially

- We estimate the value for the next period ( $Y_{t+1}$ ) as

$$\hat{Y}_{t+1} = \delta + \hat{\phi}_1 Y_t + \hat{\theta}_1 e_t$$

and then use that value to estimate the next period ( $Y_{t+2}$ )

$$\hat{Y}_{t+2} = \delta + \hat{\phi}_1 \hat{Y}_{t+1} + \hat{\theta}_1 \hat{e}_{t+1}$$

and so on

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- Let's assume a simple ARMA(1,1) model

- $Y_t = \delta + \phi_1 Y_{t-1} + \theta_1 e_{t-1} + e_t$

- We can estimate future values of  $Y$

- This is done sequentially

- We estimate the value for the next period ( $Y_{t+1}$ ) as

$$\hat{Y}_{t+1} = \delta + \hat{\phi}_1 Y_t + \hat{\theta}_1 e_t$$

and then use that value to estimate the next period ( $Y_{t+2}$ )

$$\hat{Y}_{t+2} = \delta + \hat{\phi}_1 \hat{Y}_{t+1} + \hat{\theta}_1 \hat{e}_{t+1}$$

and so on

- The uncertainty of our predictions will grow as we move into the future, away from  $t$

## Detecting Causal Effects

- We can also use time-series analysis to detect the causal effect of discrete interventions
  - I.e. policies/events that take place at a specific date
  - We can assess whether the time-series changes its properties after the intervention took place
  - Ex.1: Did the 2018 minimum wage increase in Spain had an impact on the unemployment rate?
  - Ex.2: Did the new sentencing guidelines increased sentence severity in England and Wales?
  - Often referred as interrupted time-series models

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  - Ex.1: Did the 2018 minimum wage increase in Spain had an impact on the unemployment rate?
  - Ex.2: Did the new sentencing guidelines increased sentence severity in England and Wales?
  - Often referred as interrupted time-series models
- For these types of analyses we divide the time-series in two parts
  - We model the start of the time-series up to the last time point before the intervention took place
  - Based in that model, we predict values for time periods following the intervention
  - Then we compare the predicted against the observed values



# Detecting Causal Effects

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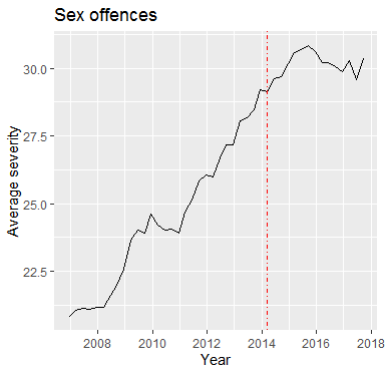
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Source: Pina-Sánchez et al. 2019



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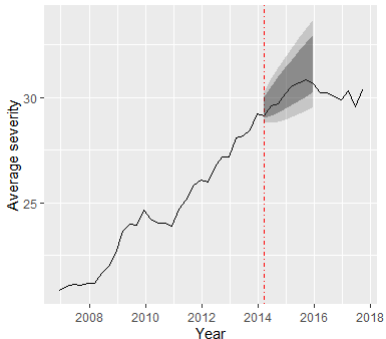
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Sex offences



Source: Pina-Sánchez et al. 2019



# Recap

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**Recap**

- We have learnt about...
  - The main components defining time-series ( $T$ ,  $S$ ,  $C$ , and  $I$ )
  - The key statistics and properties to be considered in modelling time-series (stationarity, ACF, and PACF)
  - The main family of models for the analysis of time-series (ARIMA)



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  - The main family of models for the analysis of time-series (ARIMA)
- There is so much we have not covered though
  - Exponential-smoothing methods
  - Properly modelling seasonal and cyclical effects
  - And much more

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  - The main family of models for the analysis of time-series (ARIMA)
- There is so much we have not covered though
  - Exponential-smoothing methods
  - Properly modelling seasonal and cyclical effects
  - And much more
- Recommended readings
  - Lots of free tutorials, short courses and handbooks covering time-series online
  - Hanck et al. (2019) Chapter 14 ‘Introduction to Time Series Regression and Forecasting’