Social Statistics Seminar Series:

Adjustment of Recall Errors in Duration Data Using SIMEX

Jose Pina-Sánchez

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- In addition, retrospective questions are prone to recall errors

Implications

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$$\beta_2^* = \beta_2 + \beta_1 (1 - \rho') \Gamma$$

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- Complexity of methods

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- *"The key idea underlying SIMEX is the fact that the effect of measurement error on an estimator can be determined experimentally via simulation"*

  (Carroll et al, 2006, p.98)
- I proceed to show how that is done for the case of $X^* = X + U$ in $Y = \beta_0 + \beta_1 X^* + \epsilon$

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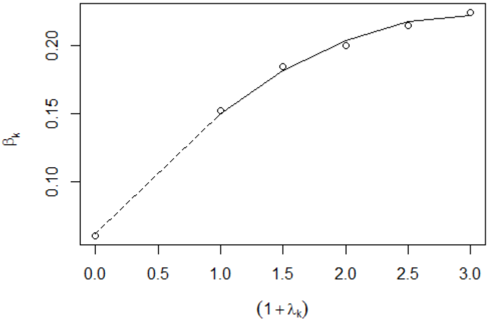
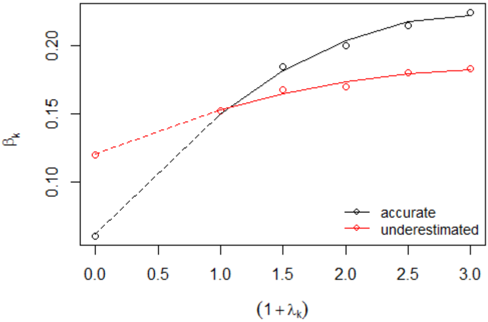
6. $\hat{\beta}_{SIMEX}$ can now be calculated by extrapolating to $\lambda_k = -1$


0.10
0.15
0.20
βk
0.0
0.5
1.0
1.5
2.0
2.5
3.0
(1+λk)


0.10
0.15
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0.0
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- Multiplicative models are elegant solutions; $X^* = X \cdot U$
- Can also be used to model telescoping effects: for forward telescoping $U \sim N(< 1, S_U^2)$, for backward telescoping $U \sim N(> 1, S_U^2)$

Multiplicative SIMEX

We substitute step 1,

$$X_k^*(\lambda_k) = X^* + \sqrt{(\lambda_k)}U$$

by either:

$$X_k^*(\lambda_k) = exp\left\{log(X^*) + \sqrt{(\lambda_k)}log(U)\right\}$$

(Carroll et al, 2006)

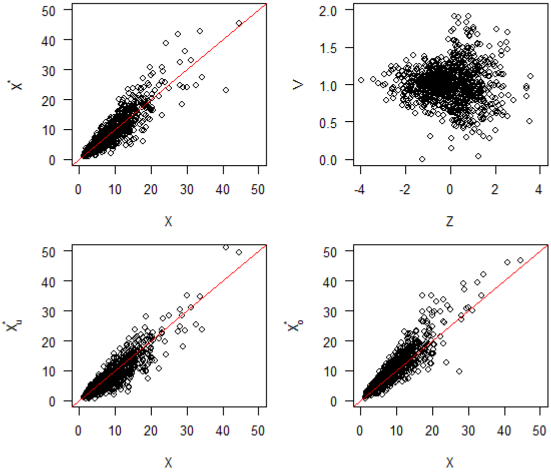
$$X_k^*(\lambda_k) = X^* \cdot U^{\lambda_k}$$

(Biewen et al., 2008)

Simulations

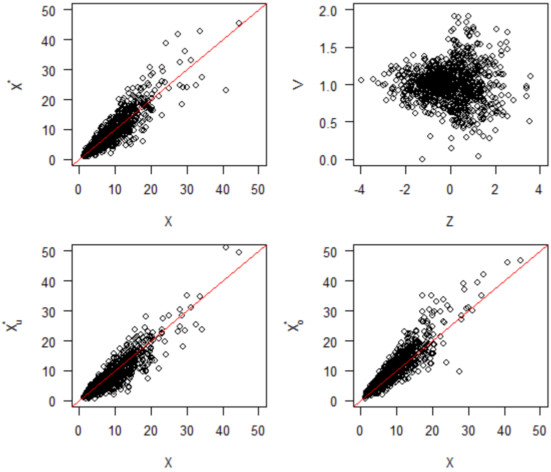
Variable	Mean	SD	Min	Max
Y	.01	.99	-3.01	3.54
Y_{ca}	.5	.5	0	1
Y_{co}	.69	.82	0	4
Z	-.02	1.1	-3.97	3.56
X	8.75	5.23	1.17	44.45

Simulated errors



Simulated errors

$U \sim N(1, .25)$



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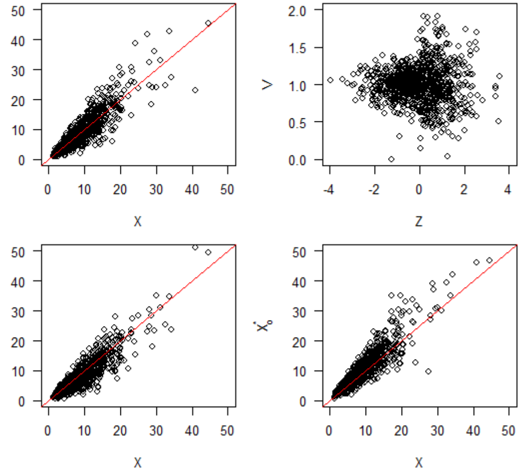

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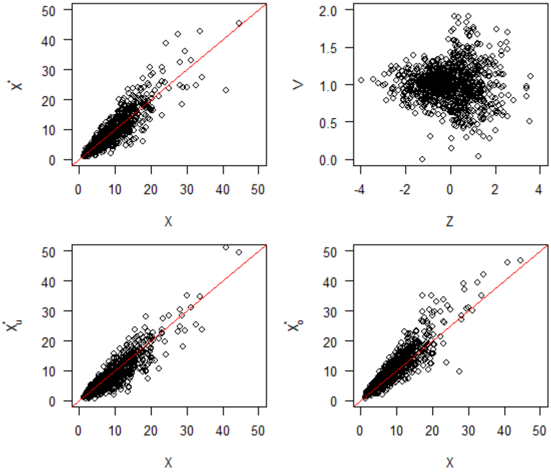

$U \sim N(.9, .25)$

$U \sim N(1.1, .25)$

Simulated errors

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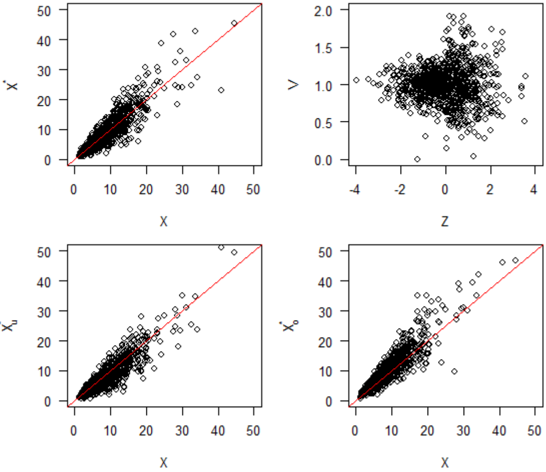
if $Z < 0$
$U \sim N(1, .30)$

if $Z > 0$
$U \sim N(1, .15)$



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Outcome models

Linear

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \epsilon$$

Logit

$$\mu = E(Y^{ca})$$

$$log(\frac{\mu}{1-\mu}) = \beta_0 + \beta_1 X + \beta_2 Z$$

Poisson

$$\mu = E(Y^{co})$$

$$log(\mu) = \beta_0 + \beta_1 X + \beta_2 Z$$

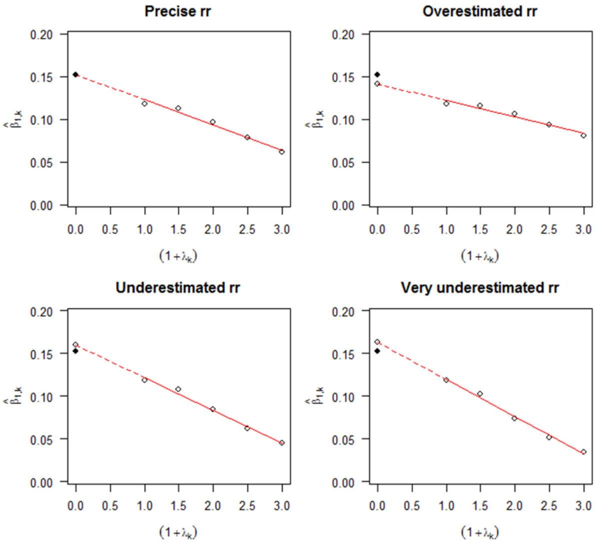
Impact

		Linear				Logit				Poisson			
		Coef	SE	Bias	R.Bias	Coef	SE	Bias	R.Bias	Coef	SE	Bias	R.Bias
True model	β_0	-1.297	.035			-5.997	.388			-1.362	.069		
	β_1	.150	.003			.768	.050			.092	.004		
	β_2	.111	.016			.210	.099			.082	.038		
Naïve: multi.	β_0	-1.013	.038	.284	21.9%	-3.810	.258	2.187	36.5%	-1.198	.065	.284	20.8%
	β_1	.118	.004	-.032	21.2%	.494	.034	-.275	37.5%	.075	.004	-.032	34.6%
	β_2	.156	.019	.045	40.9%	.275	.084	.065	30.9%	.157	.037	.045	55.1%
Naïve: hetero.	β_0	-.998	.039	.372	28.7%	-3.649	.248	2.372	39.5%	-1.178	.065	.372	27.4%
	β_1	.116	.004	-.043	28.8%	.471	.032	-.299	38.9%	.074	.004	-.043	47.0%
	β_2	.169	.020	.067	60.7%	.377	.085	.199	94.8%	.141	.037	.067	81.7%
Naïve: under.	β_0	-.937	.038	.325	25.1%	-3.441	.233	1.962	32.7%	-1.054	.059	.325	23.9%
	β_1	.121	.004	-.024	15.9%	.490	.033	-.183	23.8%	.069	.004	-.024	26.1%
	β_2	.166	.020	.052	46.9%	.321	.084	.124	58.9%	.149	.037	.052	63.2%
Naïve: over.	β_0	-1.028	.038	.245	18.9%	-4.029	.266	1.842	30.7%	-1.110	.061	.245	18.0%
	β_1	.108	.003	-.039	26.1%	.470	.031	-.284	37.0%	.061	.003	-.039	42.7%
	β_2	.150	.019	.053	48.0%	.284	.087	.152	72.4%	.134	.037	.053	64.6%

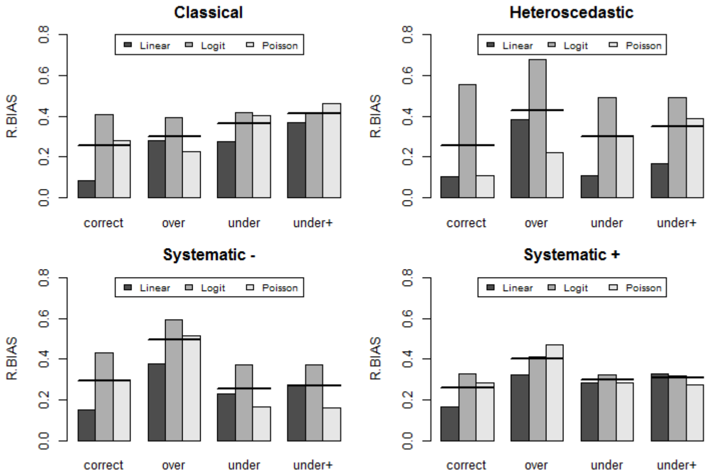
Adjustments

Scenarios	ρ	S_U^2
Correct	.816	.25
Overestimated	.9	.176
Underestimated	.7	.336
Very Underestimated	.6	.412

Results: linear model



Results


Classical
Heteroscedastic
Systematic -
Systematic +
Linear
Logit
Poisson
R.BIAS
0.0
0.2
0.4
0.6
0.8
correct
over
under
under+

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  - In all of the 144 estimates studied here SIMEX managed to reduce the bias found in the naive model by at least 22.6%
  - There are commands to run standard SIMEX in STATA and R, incorporating extensions for multiplicative errors would make it even easier for everyone to consider sensitivity analysis when using retrospective data.